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Coupled interpolation-collocation for off mesh approximation of numerical solutions of PDEs

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Abstract

In this paper we address the problem of accurate and efficient interpolation of numerical solutions of partial differential equations (PDE)s. This problem arises frequently, for example, with the static adaptive method of lines (MOL) where a numerical solution from a previous grid needs to be mapped accurately onto a new adapted grid or when accurate and efficient interpolations at off mesh points is required for fast post-processing, (such as post-evaluations of additional dependent variables, visualization and animation), of a subset of numerical results of a large scale simulation or numerical results related only to a specific region of interest of the whole computational domain. In most previous studies the approximations at off-meshes are performed using cubic or bicubic splines interpolations but these interpolation techniques are not coupled to the PDE and can be inefficient and insufficiently accurate for high order PDEs such as Korteweg-de Vries equations and Boussinesq equations. In this paper, we first present a review of standard interpolating algorithms (based on splines interpolations and Hermite interpolations and then we introduce a new interpolation procedure proposed recently by Enright (2000) as an alternative to traditional interpolation methods. This new interpolation, called Differential Equation Interpolation (DEI), is based on associating a set of collocation points with each mesh element and requiring that the local approximation interpolates the meshpoint data and almost satisfies the PDE at the collocation points. It is shown that the accuracy of this interpolation, which is coupled to collocation, is higher than the accuracy of traditional interpolations and consistent with the mesh point accuracy of the MOL solution. Accurate off-mesh approximations of the numerical solution is required for the assessment of the performance of the solution method of evolution PDEs that model nonlinear dispersive waves. In such cases, the EDI provides accurate evaluation of the phase and amplitude errors and accurate integration of invariants of motion which are used to measure the reliability of the underlying solution method of the PDE. The EDI applies to a large class of PDEs (elliptic, parabolic and hyperbolic) including linear and nonlinear problems. It can also be applied directly to first order, second order, high order and mixed order PDEs. Numerical examples are given for some high order, mixed order and nonlinear problems in one, two and three dimensions to illustrate important features of the EDI.